

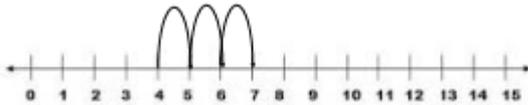
ADDITION

1. Practical activities using practical resources and pictures e.g. small toys, blocks



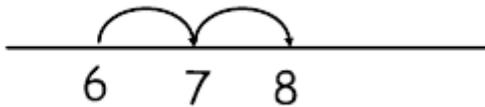
$$3 + 5 = 8$$

2. Counting on using a number line (always start on the largest number first)



$$4 + 3 = 7$$

3. Addition using an empty numberline



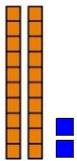
$$6 + 2 = 8$$

4. Addition of a one digit number to a two digit number using a hundred square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$$23 + 4 = 27$$

5. Partitioning a two digit number (using diennes nd numicon for understanding)



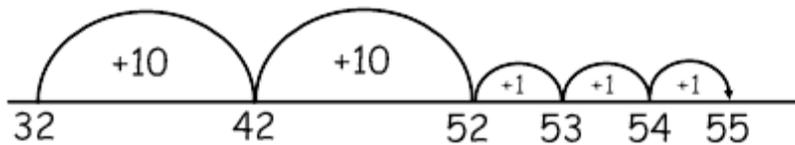
$$22 = 20 + 2$$

6. Addition of a two digit number to a two digit number by partitioning the second number using the hundred square. E.g. $32 + 23 = 55$, $32 + 20 + 3 = 55$. Begin on 32, go down the column 2 squares (20) and then forwards along the row 2 squares.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

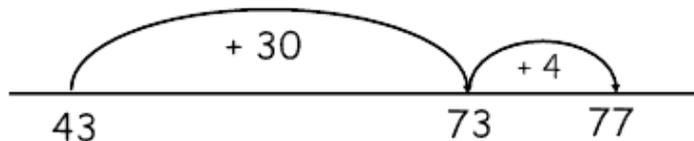
$$32 + 23 = 55$$

7. Then using an empty number line:



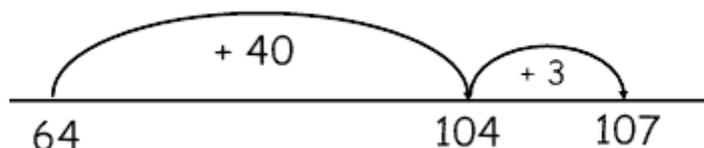
$$32 + 23 = 55$$

8. Then using more efficient jumps:



$$43 + 34 = 77$$

9. Then with numbers which involve bridging:



$$64 + 43 = 107$$

Practical resources and equipment can be used for all of the addition methods for up to this point. These earlier methods are to secure the understanding of addition. **The expectation for the end of Year 2 is to add two two-digit numbers using concrete objects, pictorial representations and mentally.**

10. Informal Written Method (Horizontal)

It is really important to model to the children how to set out their workings and make sure they are leaving two squares between numbers when setting it out.

Make sure you start with 2 digit + digit with numbers which do not bridge:

e.g. $43 + 25$

	4	0		3
+	2	0		5
<hr/>				
				8

add the units first

	4	0		3
+	2	0		5
<hr/>				
	6	0		8

add the tens next

4	0		3		
+	2	0	5		
<hr/>					
6	0	+	8	=	6 8

add both totals for final answer

11. Then same method but with up to 3 digit numbers with numbers which do bridge:

$$354 + 575 = 929$$

3	0	0		5	0		4			
+	5	0	0		7	0		5		
<hr/>										
8	0	0	+	1	2	0	+	9	=	9 2 9

12. Expanded Written Method (Vertical)

The same method can be used in a vertical calculation with the smallest parts of the numbers being added first and the largest parts of the numbers added last. It is now vital that children keep digits in the correct columns.

e.g. $36 + 42$

	T	U
	3	6
+	4	2
<hr/>		
		8

Add the units first by saying six plus two

	T	U
	3	6
+	4	2
<hr/>		
		8
	7	0

Add the tens by saying thirty plus forty is seventy

	T	U
	3	6
+	4	2
<hr/>		
		8
	7	0
<hr/>		
	7	8

Total the numbers. How many units do we have? Eight. How many tens do we have? Seven tens so the answer is 78.

Note: You can then move onto numbers which bridge as long as you don't need to bridge again when you are adding your totals as we are not introducing carrying until the next step in the method.

13. Larger number which involve bridging:

e.g. $224 + 567$

	H	T	U
	2	2	4
+	5	6	7
<hr/>			
		1	1
		8	0
	7	0	0
<hr/>			
	7	9	1

14. Standard Compact Written Method (Vertical & Compact)

This can then lead to a more compact method involving carrying between columns where necessary. You must start with smaller numbers until the children understand the concept of carrying.

e.g. $38 + 25$

	T	U
	3	8
+	2	5
<hr/>		
		3
<hr/>		
	1	

Add the units.

Eight plus five is thirteen.

Put one ten under the tens column and four in the units column.

	T	U
	3	8
+	2	5
<hr/>		
	6	3
<hr/>		
	1	

Add the tens.

Thirty plus twenty plus ten underneath is sixty.

Note: make sure children cross off any numbers they carry as they go along.

15. You can then move children onto the same method with bigger numbers:

e.g. $463 + 459$

	H	T	U
	4	6	3
+	4	5	9
<hr/>			
	9	2	2
	↖	↖	

This is the expectation for the end of Year 3. At the end of year 4, children should be able to add two four-digit numbers.

16. Then move onto adding money amounts, making sure children put the decimal point in straight away:

e.g. $£1.56 + £6.73$

	1	.	5	6
+	6	.	7	3
<hr/>				
	8	.	2	9
	↖			

17. Finally, children should be able to add amounts up with different numbers of decimal places:

e.g. $9.835 + 5.7$

	9	.	8	3	5	
+	5	.	7	0	0	
<hr/>						
	1	5	.	5	3	5
	↖	↖				

Note: Children should add 0s in to the place holders so that both numbers have the same amount of decimal places to help.

At the end of Year 5, children are expected to solve problems with numbers with up to 3 decimal places.

Subtraction

1. Practical activities using practical resources and pictures e.g. small toys or blocks



4 fish



2 swam away

How many are left?

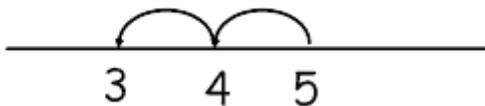
$$4 - 2 = 2$$

2. Counting back using a number line



$$6 - 2 = 4$$

3. Subtraction using an empty number line



$$5 - 2 = 3$$

4. Subtraction of a one digit number from a two digit number using a hundred square

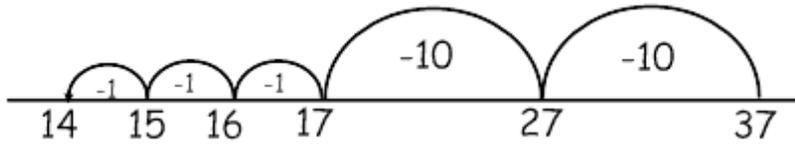
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$$45 - 4 = 41$$

5. Subtraction of a two digit number from a two digit number by partitioning the second number using a hundred square. E.g. $37 - 23 = 14$, $37 - 20 - 3 = 14$. Begin on 37, go up the column 2 squares (20) then backwards along the row 3 squares.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

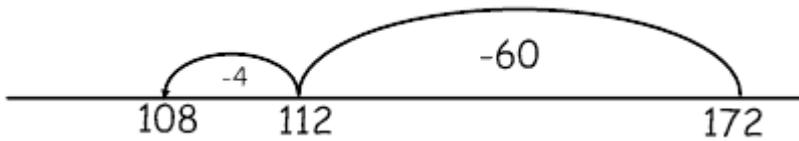
6. Then using an empty number line



$$37 - 23 = 14$$

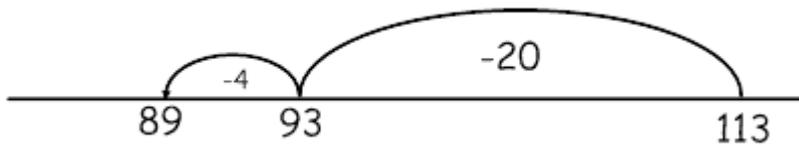
The expectation for the end of Year 2 is to subtract two two-digit numbers using concrete objects, pictorial representations and mentally.

7. Using an empty number line for bigger numbers (up to 3 digit – 3 digit) using more efficient jumps



$$172 - 64 = 108$$

8. Using an empty number line involving bridging a hundred



$$113 - 24 = 89$$

9. Expanded Written Method – Decomposition

We can also use ideas of partitioning to take away when subtracting. This method partitions each number and takes each part of one number away from each part of the other number. This can be introduced at first with two digit numbers which do not involve decomposition.

$$37 - 24 =$$

30		7		
- 20		4		
<hr/>				
10	+	3	=	13

10. Once confident with 2 digits, move onto 3 digits:

$$286 - 172 =$$

200		80		6		
+ 100		70		2		
<hr/>						
100	+	10	+	4	=	114

11. Once children have an understanding of how it is set out and the place value, they can then move onto calculations which involve decomposition. This should be modelled to them with the visual aid of diennes.

$$42 - 18 =$$

	30					
	40		¹ 2			
-	10		8			
<hr/>						
	20	+	4	=	24	

Once they have a good understanding of decomposition, the diennes can then be taken away to solve similar calculations and then move on to three digits minus three digits.

12. Standard Compact Written Method

This expanded written method then leads to a more compact method. Children should start with 2 digits minus 2 digit and then progress to bigger numbers (up to 4 digits).

$$83 - 27 =$$

	7	
	8	¹ 3
-	5	6
<hr/>		
	2	7

13. Progress to questions where children cannot borrow from the next column but have to borrow from 2 columns along:

$$302 - 117$$

	2	9	
	3	0	¹ ¹ 2
-	1	1	7
<hr/>			
	1	8	5

This is the expectation for the end of Year 3. At the end of year 4, children should be able to subtract two four-digit numbers.

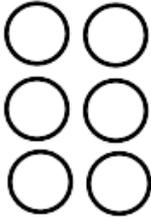
14. Children can then use the same method to progress to larger numbers, money amounts and decimals with numbers with different numbers of decimal points.

At the end of Year 5, children are expected to solve problems with numbers with up to 3 decimal places.

MULTIPLICATION

1. Using arrays for a concrete understanding of what multiplication is:

$$3 \times 2$$



Teachers should explain that this is 3 lots of 2 but also 2 lots of 3 (commutative).

This is the expectation for the end of Year 2. Children should also know then 2, 5 and 10 times tables.

2. Times tables facts. **The expectation for the end of Year 3 is to know 2, 5, 10, 3, 4 and 8 times tables. The expectation for the end of Year 4 is to know all of their times tables up to 12.**
3. Multiplying by 10 using equipment (diennes, numicon, place value grids) to help with the concept initially.
4. Multiplying up multiples of 10 using what they know. So $3 \times 2 = 6$ so $3 \times 20 = 60$
5. Formal method using partitioning. Children need to partition the larger number and multiply each column separately. They then need to use their addition method to add the totals.

$$27 \times 8 =$$

	2	0		7		8	0	
x				4	+	2	8	
	8	0	2	8		1	0	8

6. Expanded written method. **This is the end of Year 3 expectation according to the new curriculum with 2 digit numbers.** They should only be using the times tables they know (2, 3, 4, 5, 8). Children should be using jottings by the side to help with their understanding of the method.

	2	3		
x		4		
	1	2	3 × 4	
	8	0	20 × 4	
	9	2		

7. Expanded written method multiplying 3 digit numbers by 1 digit. They should first of multiply the ones ($4 \times 5 = 20$), then the tens column ($30 \times 5 = 150$) and then the hundreds column ($200 \times 5 = 1000$). Finally they should add up their three answers using their addition method to find the final answer.

	2	3	4		
x			5		
		2	0	4 × 5	
	1	5	0	30 × 5	
	1	0	0	200 × 5	
	1	1	7	0	

Note: For this method, children need to be very confident multiplying numbers by both multiples of 10 and 100.

8. Compact formal method. Children should begin once again by multiplying a 2 digit number by a 1 digit number until confident. When modelling, you need to keep reinforcing place value.

	T	U
	1	7
x		4
	6	8
	2	

Then they can move onto 3 digit number by 1 digit. **This is the expectation for the end of Year 4.**

9. Using the compact formal method for multiplying 2 digit numbers by 2 digit numbers (with the second number to have 1 in the tens) *Note: When using this method, children should always multiply by the units column first so they are in the habit of working from right to left through the columns.*

	5	2	
x	1	4	
<hr/>			
5	2	0	
2	0	8	
<hr/>			
7	2	8	

Note: Smaller numbers to be crossed off in the middle section so children don't get confused when adding their columns for the final answer.

Then you can use the compact formal method for multiplying a range of 2 digit numbers by **By the end of Year 5, children are expected to use the formal method to multiply numbers up to 4 digits by 1 digit and 2 digit numbers.**

10. 2 digits and then move onto 3 digit by 2 digit:

	2	3	6	
x		2	4	
<hr/>				
	9	4	4	
4	7	2	0	
<hr/>				
5	6	6	4	

11. Children can be extended to multiply decimal amounts. It is important that the numbers are still written in the correct place value columns.

	2	3	.	0	7
x		4			
<hr/>					
	9	2	.	2	8

This is the expectation for the end of Year 6.

DIVISION

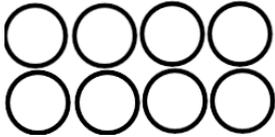
- Using arrays for a concrete understanding of what division is:

$$6 \div 3$$



Teachers should explain that this shows $6 \div 3 = 2$ but also $6 \div 2 = 3$

- Children should be confident to find fact families using arrays:



e.g. $4 \times 2 = 8$

$$2 \times 4 = 8$$

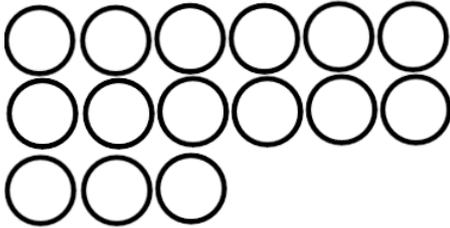
$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

This is the expectation for the end of Year 2. Children should also know then 2, 5 and 10 times tables.

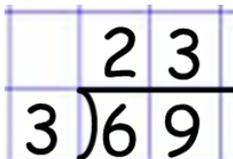
- Times tables facts. **The expectation for the end of Year 3 is to know 2, 5, 10, 3, 4 and 8 times tables and division facts for them. The expectation for the end of Year 4 is to know all of their times tables up to 12 and the division facts for them.**
- Remainders shown with arrays:

$$15 \div 6 = 2 \text{ r}3 \text{ (They are two rows of 6 and 3 left over)}$$



- A skill alongside the stages of division should be mental division of multiples – they should be using what they know.
E.g. $80 \div 4 = 20$
- Introduce bus stop method with numbers where there is no carrying and no remainder. Make sure that when modelling, you are discussing place value

e.g. $69 \div 3 = 23$



7. Same but this time with a remainder:

e.g. $87 \div 4 = 21 \text{ r}3$

	2	1	r	3
4)	8	7	

This is the expectation for the end of Year 3.

8. Then, introduce simple carrying:

e.g. $75 \div 5 = 15$

	1	5
5)	7 ² 5

9. Move onto larger numbers:

e.g. $256 \div 4 = 64$

	0	6	4
4)	2 ² 5 ¹ 6	

10. Larger numbers with remainders:

e.g. $567 \div 4 = 141 \text{ r}3$

	1	4	1	r	3
4)	5 ¹ 6 ¹ 7			

Note: Children are expected to know how to show their remainder as a fraction too. So the answer could also be written as $141 \frac{3}{4}$

11. Dividing money amounts:

e.g. $\pounds 14.65 \div 5 = \pounds 2.93$

		0	2	.	9	3
5)	1	¹ 4	.	⁴ 6	¹ 5

This is the expectation for the end of Year 5.

12. Showing a remainder as a decimal amount:

e.g. $457 \div 8 =$

		0	5	7	.	1	2	5
8)	4	⁴ 5	⁵ 7	.	¹ 0	² 0	⁴ 0

13. Using long division to divide by a 2 digit number:

						15
			2	8		30
1	5		4	3	2	45
					r12	60
			3	0	0	75
			1	3	2	90
			1	2	0	105
						120
			1	2		135
						150

Note: children should be encouraged to write down their jottings to help them.

This is the expectation for the end of Year 6.